



AP[®] Calculus AB

Sample Multiple-Choice Questions

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THIS IS NOT A COMPLETE EXAMINATION.

Part A Sample Multiple-Choice Questions

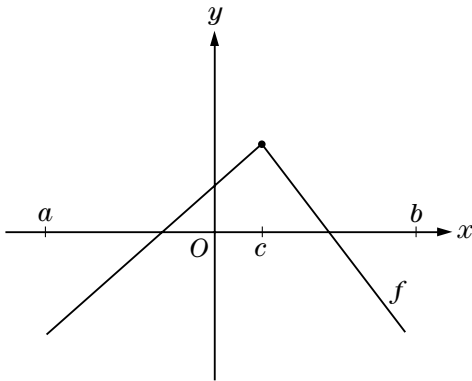
A calculator may not be used on this part of the examination.

Part A consists of 28 questions. In this section of the examination, as a correction for guessing, one-fourth of the number of questions answered incorrectly will be subtracted from the number of questions answered correctly. Following are the directions for Section I Part A and a representative set of 14 questions.

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1.



The function f , whose graph consists of two line segments, is shown above. Which of the following are true for f on the open interval (a, b) ?

- I. The domain of the derivative of f is the open interval (a, b) .
- II. f is continuous on the open interval (a, b) .
- III. The derivative of f is positive on the open interval (a, c) .

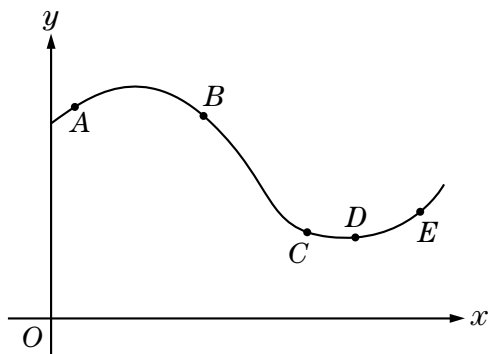
- (A) I only
- (B) II only
- (C) III only
- (D) II and III only
- (E) I, II, and III

2. What is $\lim_{h \rightarrow 0} \frac{\cos\left(\frac{\pi}{2} + h\right) - \cos\left(\frac{\pi}{2}\right)}{h}$?

- (A) -1
- (B) $-\frac{\sqrt{2}}{2}$
- (C) 0
- (D) 1
- (E) The limit does not exist.

3. At which of the five points on the graph in the figure at the right are $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ both negative?

- (A) A
 (B) B
 (C) C
 (D) D
 (E) E



4. The slope of the tangent to the curve $y^3x + y^2x^2 = 6$ at $(2, 1)$ is

- (A) $-\frac{3}{2}$
 (B) -1
 (C) $-\frac{5}{14}$
 (D) $-\frac{3}{14}$
 (E) 0

5. Which of the following statements about the function given by $f(x) = x^4 - 2x^3$ is true?

- (A) The function has no relative extremum.
 (B) The graph of the function has one point of inflection and the function has two relative extrema.
 (C) The graph of the function has two points of inflection and the function has one relative extremum.
 (D) The graph of the function has two points of inflection and the function has two relative extrema.
 (E) The graph of the function has two points of inflection and the function has three relative extrema.

6. If $f(x) = \sin^2(3 - x)$, then $f'(0) =$

- (A) $-2 \cos 3$
 (B) $-2 \sin 3 \cos 3$
 (C) $6 \cos 3$
 (D) $2 \sin 3 \cos 3$
 (E) $6 \sin 3 \cos 3$

7. The solution to the differential equation $\frac{dy}{dx} = \frac{x^3}{y^2}$, where $y(2) = 3$, is

(A) $y = \sqrt[3]{\frac{3}{4}x^4}$

(B) $y = \sqrt[3]{\frac{3}{4}x^4} + \sqrt[3]{15}$

(C) $y = \sqrt[3]{\frac{3}{4}x^4} + 15$

(D) $y = \sqrt[3]{\frac{3}{4}x^4 + 5}$

(E) $y = \sqrt[3]{\frac{3}{4}x^4 + 15}$

8. $\int (x - 1)\sqrt{x} \, dx =$

(A) $\frac{3}{2}\sqrt{x} - \frac{1}{\sqrt{x}} + C$

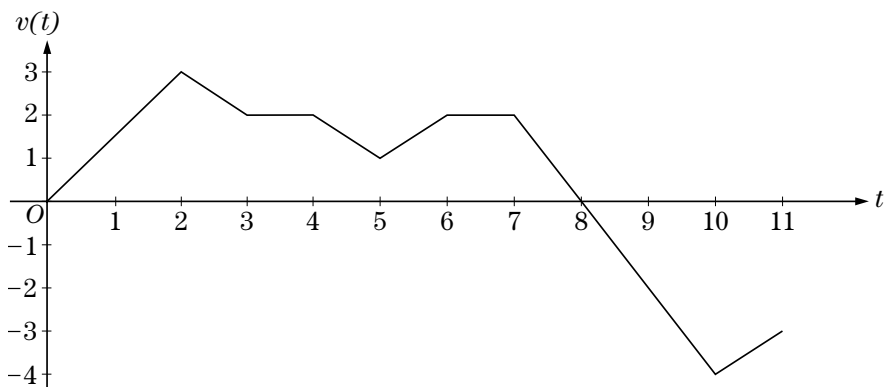
(B) $\frac{2}{3}x^{\frac{3}{2}} + \frac{1}{2}x^{\frac{1}{2}} + C$

(C) $\frac{1}{2}x^2 - x + C$

(D) $\frac{2}{5}x^{\frac{5}{2}} - \frac{2}{3}x^{\frac{3}{2}} + C$

(E) $\frac{1}{2}x^2 + 2x^{\frac{3}{2}} - x + C$

Questions 9 and 10 refer to the following graph and information.



A bug is crawling along a straight wire. The velocity, $v(t)$, of the bug at time t , $0 \leq t \leq 11$, is given in the graph above.

9. According to the graph, at what time t does the bug change direction?
- (A) 2
(B) 5
(C) 6
(D) 8
(E) 10
10. According to the graph, at what time t is the speed of the bug greatest?
- (A) 2
(B) 5
(C) 6
(D) 8
(E) 10

11. What is $\lim_{x \rightarrow \infty} \frac{x^2 - 4}{2 + x - 4x^2}$?
- (A) -2
(B) $-\frac{1}{4}$
(C) $\frac{1}{2}$
(D) 1
(E) The limit does not exist.
12. The area of the region in the first quadrant between the graph of $y = x\sqrt{4 - x^2}$ and the x -axis is
- (A) $\frac{2}{3}\sqrt{2}$
(B) $\frac{8}{3}$
(C) $2\sqrt{2}$
(D) $2\sqrt{3}$
(E) $\frac{16}{3}$
13. Which of the following are antiderivatives of $\frac{\ln^2 x}{x}$?
- I. $\frac{\ln^3 x}{3}$
II. $\frac{\ln^3 x}{3} + 6$
III. $\frac{2 \ln x - \ln^2 x}{x^2}$
- (A) I only
(B) III only
(C) I and II only
(D) I and III only
(E) II and III only

14. If r is positive and increasing, for what value of r is the rate of increase of r^3 twelve times that of r ?
- (A) $\sqrt[3]{4}$
 - (B) 2
 - (C) $\sqrt[3]{12}$
 - (D) $2\sqrt{3}$
 - (E) 6

Part B Sample Multiple-Choice Questions

A graphing calculator is required for some questions on this part of the examination.

Part B consists of 17 questions. In this section of the examination, as a correction for guessing, one-fourth of the number of questions answered incorrectly will be subtracted from the number of questions answered correctly. Following are the directions for Section I Part B and a representative set of 10 questions.

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test:

- (1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.
 - (2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.
15. The average value of the function $f(x) = e^{-x^2}$ on the closed interval $[-1, 1]$ is
- (A) 0
 - (B) 0.368
 - (C) 0.747
 - (D) 1
 - (E) 1.494

16. The volume generated by revolving about the x -axis the region enclosed by the graphs of $y = 2x$ and $y = 2x^2$, for $0 \leq x \leq 1$, is

(A) $\pi \int_0^1 (2x - 2x^2)^2 dx$

(B) $\pi \int_0^1 (4x^2 - 4x^4) dx$

(C) $2\pi \int_0^1 x(2x - 2x^2) dx$

(D) $\pi \int_0^2 \left(\sqrt{\frac{y}{2}} - \frac{y}{2} \right)^2 dy$

(E) $\pi \int_0^2 \left(\frac{y}{2} - \frac{y^2}{2} \right) dy$

17. Let f be defined as follows, where $a \neq 0$.

$$f(x) = \begin{cases} \frac{x^2 - a^2}{x - a}, & \text{for } x \neq a, \\ 0, & \text{for } x = a. \end{cases}$$

Which of the following are true about f ?

- I. $\lim_{x \rightarrow a} f(x)$ exists.
 - II. $f(a)$ exists.
 - III. $f(x)$ is continuous at $x = a$.
- (A) None
(B) I only
(C) II only
(D) I and II only
(E) I, II, and III

18.

x	1.1	1.2	1.3	1.4
$f(x)$	4.18	4.38	4.56	4.73

Let f be a function such that $f''(x) < 0$ for all x in the closed interval $[1, 2]$, with selected values shown in the table above. Which of the following must be true about $f'(1.2)$?

- (A) $f'(1.2) < 0$
- (B) $0 < f'(1.2) < 1.6$
- (C) $1.6 < f'(1.2) < 1.8$
- (D) $1.8 < f'(1.2) < 2.0$
- (E) $f'(1.2) > 2.0$

19. Two particles start at the origin and move along the x -axis. For $0 \leq t \leq 10$, their respective position functions are given by $x_1 = \sin t$ and $x_2 = e^{-2t} - 1$. For how many values of t do the particles have the same velocity?

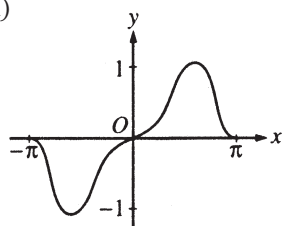
- (A) None
- (B) One
- (C) Two
- (D) Three
- (E) Four

20. If the function g is defined by $g(x) = \int_0^x \sin(t^2) dt$ on the closed interval $-1 \leq x \leq 3$, then g has a local minimum at $x =$

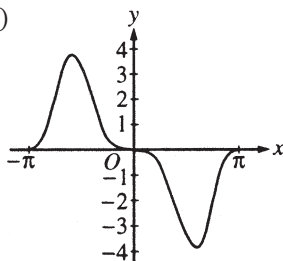
- (A) 0
- (B) 1.084
- (C) 1.772
- (D) 2.171
- (E) 2.507

21. The graphs of five functions are shown below. Which function has a nonzero average value over the closed interval $[-\pi, \pi]$?

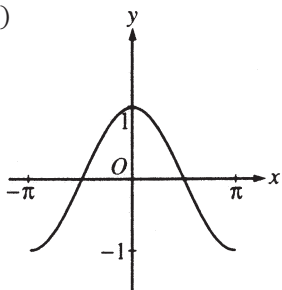
(A)



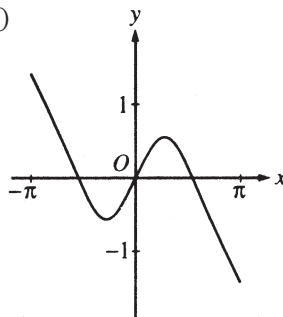
(B)



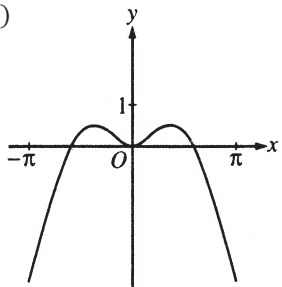
(C)



(D)



(E)



22. The region in the first quadrant enclosed by the y -axis and the graphs of $y = \cos x$ and $y = x$ is rotated about the x -axis. The volume of the solid generated is
- (A) 0.484
(B) 0.877
(C) 1.520
(D) 1.831
(E) 3.040
23. Oil is leaking from a tanker at the rate of $R(t) = 2,000e^{-0.2t}$ gallons per hour, where t is measured in hours. How much oil has leaked out of the tanker after 10 hours?
- (A) 54 gallons
(B) 271 gallons
(C) 865 gallons
(D) 8,647 gallons
(E) 14,778 gallons
24. If f is continuous for all x , which of the following integrals necessarily have the same value?
- I. $\int_a^b f(x)dx$
II. $\int_0^{b-a} f(x+a)dx$
III. $\int_{a+c}^{b+c} f(x+c)dx$
- (A) I and II only
(B) I and III only
(C) II and III only
(D) I, II, and III
(E) No two necessarily have the same value.

Answers to Calculus AB Multiple-Choice Questions

Part A

- | | | |
|------|-------|-------|
| 1. D | 6. B | 11. B |
| 2. A | 7. E | 12. B |
| 3. B | 8. D | 13. C |
| 4. C | 9. D | 14. B |
| 5. C | 10. E | |

Part B

- | | | |
|--------|--------|--------|
| 15.* C | 19.* D | 22.* C |
| 16. B | 20.* E | 23.* D |
| 17. D | 21. E | 24. A |
| 18. D | | |

* Indicates a graphing calculator-active question.